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RADICALES

1.- Simplifica los siguientes radicales:

$\sqrt[12]{x^9} = \sqrt[3\cdot4]{x^{3\cdot3}} = \sqrt[4]{x^3}$

$\sqrt[5]{y^{10}} = \sqrt[5\cdot1]{x^{5\cdot2}} = y^2$

$\sqrt[8]{x^6} = \sqrt[2\cdot4]{x^{2\cdot3}} = \sqrt[4]{x^3}$

$\sqrt[8]{16} = \sqrt[2\cdot4]{2^{2\cdot2}} = \sqrt[2]{2}$

$\sqrt[9]{64} = \sqrt[3\cdot3]{2^{3\cdot3}} = 2$

2.- Reducir a índice común:

$$\sqrt[12]{x^5}\, y \sqrt[18]{x^7} \Rightarrow \begin{array}{l} \sqrt[12]{x^5} = \sqrt[12\cdot3]{x^{7\cdot3}} = \sqrt[36]{x^{21}} \\ \sqrt[18]{x^7} = \sqrt[18\cdot2]{x^{7\cdot2}} = \sqrt[36]{x^{14}} \end{array}$$

$$\sqrt[3]{46}\, y \sqrt[9]{132100} \Rightarrow \begin{array}{l} \sqrt[3]{46} = \sqrt[3\cdot3]{46^{1\cdot3}} = \sqrt[9]{97336} \\ \sqrt[9]{132100} = \sqrt[9]{132100} \end{array}$$

3.- Simplifica los siguientes radicales:

$$\left(\sqrt{\sqrt{\sqrt[3]{x}}}\right)^8 = \sqrt[12]{x^8} = x^{\frac{8}{12}} = x^{\frac{2}{3}} = \sqrt{x^3}$$

$$\left(\sqrt{\sqrt[5]{\sqrt{x}}}\right)^{12} = \sqrt[20]{x^{12}} = x^{\frac{12}{20}} = x^{\frac{3}{5}} = \sqrt[3]{x^5}$$

$$\sqrt[3]{\sqrt{32}} = \sqrt[6]{2^5}$$

$$\sqrt{2\sqrt{3\sqrt{4}}} = \sqrt[6]{24}$$

$$\sqrt[3]{3\sqrt[3]{\frac{1}{9}}} = \sqrt[9]{\frac{3}{9}} = \sqrt[9]{\frac{1}{3}} = \sqrt[9]{3^{-1}}$$

$$3\sqrt{3\sqrt{\frac{1}{3}\sqrt{3^3}}} = 3\sqrt[8]{\frac{3^4}{3}} = 3\sqrt[8]{3^3}$$

4a.- Multiplicación de radicales

$$\sqrt{12}\cdot\sqrt[3]{36}=\sqrt[6]{12^3}\cdot\sqrt[6]{36^2}=\sqrt[6]{12^3\cdot 36^2}=\sqrt[6]{\left(2^2\cdot 3\right)^3\cdot\left(2^2\cdot 3^2\right)^2}=\sqrt[6]{2^{10}\cdot 3^7}=2\cdot 3\sqrt[6]{2^4\cdot 3}=6\sqrt[6]{48}$$

$m.c.m(2,3)=6$

$$\sqrt{9}\cdot\sqrt[5]{27}\cdot\sqrt[4]{3}=\sqrt[20]{\left(3^2\right)^{10}}\cdot\sqrt[20]{\left(3^3\right)^4}\cdot\sqrt[20]{3^5}=\sqrt[20]{3^{20}\cdot 3^{12}\cdot 3^5}=\sqrt[20]{3^{37}}=3\sqrt[20]{3^{17}}$$

$m.c.m(4,2,5)=20$

$$\sqrt{3}\cdot\sqrt[3]{9}\cdot\sqrt[4]{27}=\sqrt[12]{3^6}\cdot\sqrt[12]{\left(3^2\right)^4}\cdot\sqrt[12]{\left(3^3\right)^3}=\sqrt[12]{3^6\cdot 3^8\cdot 3^9}=\sqrt[12]{3^{23}}=3\sqrt[12]{3^{11}}$$

$m.c.m(2,3,4)=12$

$$\sqrt{3}\cdot\sqrt[4]{9}\cdot\sqrt[5]{27}\cdot\sqrt{81}=\sqrt[20]{3^{10}}\cdot\sqrt[20]{\left(3^2\right)^5}\cdot\sqrt[20]{\left(3^3\right)^4}\cdot\sqrt[20]{\left(3^4\right)^{10}}=\sqrt[20]{3^{10}\cdot 3^{10}\cdot 3^{12}\cdot 3^{40}}=\sqrt[20]{3^{72}}=3^3\cdot\sqrt[20]{3^{12}}$$

$m.c.m(4,2,5)=20$

$$\sqrt{a}\cdot\sqrt[3]{\frac{1}{a}}\cdot\sqrt[4]{a^3}=\sqrt[12]{a^6}\cdot\sqrt[12]{\left(a^{-1}\right)^4}\cdot\sqrt[12]{\left(a^3\right)^3}=\sqrt[12]{a^6\cdot a^{-4}\cdot a^9}=\sqrt[12]{a^{11}}$$

$m.c.m(2,3,4)=12$

$$\sqrt{\sqrt[3]{a}}\cdot\sqrt{\left(\frac{1}{a}\right)^2}\cdot\sqrt[4]{a^3}=\sqrt[6]{a}\cdot\sqrt{\frac{1}{a^2}}\cdot\sqrt[4]{a^3}=\sqrt[12]{a^2}\cdot\sqrt[12]{\left(a^{-2}\right)^6}\cdot\sqrt[12]{\left(a^3\right)^3}=\sqrt[12]{a^2\cdot a^{-12}\cdot a^9}=\sqrt[12]{a^{-1}}$$

$m.c.m(2,4,6)=12$

4b.- División de radicales

$$\frac{\sqrt[3]{4}}{\sqrt{2}}=\frac{\sqrt[6]{4^2}}{\sqrt[6]{2^3}}=\sqrt[6]{\frac{\left(2^2\right)^2}{2^3}}=\sqrt[6]{\frac{2^4}{2^3}}=\sqrt[6]{2}$$

$m.c.m(2,3)=6$

.

$$\frac{\sqrt{256}}{\sqrt[3]{16}} = \frac{\sqrt[6]{256^3}}{\sqrt[6]{16^2}} = \sqrt[6]{\frac{(2^8)^3}{(2^4)^2}} = \sqrt[6]{\frac{2^{24}}{2^8}} = \sqrt[6]{2^{16}} = \sqrt[3]{2^8} = 2^2\sqrt[3]{2^2} = 4\sqrt[3]{4}$$

$m.c.m(2,3) = 6$

$$\frac{\sqrt[4]{625}}{\sqrt[3]{25}} = \frac{\sqrt[12]{(5^4)^3}}{\sqrt[12]{(5^2)^4}} = \sqrt[12]{\frac{5^{12}}{5^8}} = \sqrt[12]{5^4}$$

$m.c.m(4,3) = 12$

$$\frac{\sqrt[4]{81}}{\sqrt{9}} = \frac{\sqrt[4]{(3^4)^1}}{\sqrt[4]{(3^2)^2}} = \sqrt[4]{\frac{3^4}{3^4}} = \sqrt[4]{1} = 1$$

$m.c.m(4,2) = 4$

$$\frac{\sqrt{\sqrt[4]{81}}}{\sqrt[3]{\sqrt{9}}} = \frac{\sqrt[8]{3^4}}{\sqrt[6]{3^2}} = \frac{\sqrt[24]{(3^4)^3}}{\sqrt[24]{(3^2)^4}} \sqrt[24]{\frac{3^{12}}{3^8}} = \sqrt[24]{3^4}$$

$m.c.m(6,8) = 24$

5.- Suma y resta de radicales:

$5\sqrt{x} + 3\sqrt{x} + 4\sqrt{x} = 12\sqrt{x}$

$\sqrt{125} + \sqrt{54} - \sqrt{45} + \sqrt{24} = \sqrt{5^3} + \sqrt{2\cdot3^3} + \sqrt{5\cdot3^2} + \sqrt{3\cdot2^3} = 5\sqrt{5} + 3\cdot\sqrt{3\cdot2} + 3\sqrt{5} + 3\cdot\sqrt{3\cdot2} =$
$= 8\sqrt{5} + 6\sqrt{6}$

$\sqrt{18} + \sqrt{50} + \sqrt{8} = \sqrt{2\cdot3^2} + \sqrt{5^2\cdot2} + \sqrt{2\cdot2^2} = 3\sqrt{2} + 5\sqrt{2} + 2\sqrt{2} = 10\sqrt{2}$

$\sqrt{27} + \sqrt{81} + \sqrt{18} = \sqrt{3^3} + \sqrt{3^4} + \sqrt{2\cdot3^2} = 3\sqrt{3} + 3^2 + 3\sqrt{2} = 9 + 6\sqrt{3}$

$\sqrt{125} + \sqrt{180} - \sqrt{45} = \sqrt{5^3} + \sqrt{3^2\cdot5\cdot2^2} - \sqrt{5\cdot3^2} = 5\sqrt{5} + 3\cdot2\sqrt{5} - 3\sqrt{5} = 8\sqrt{5}$

.

$$5\sqrt{125}+6\sqrt{45}-7\sqrt{20}+\frac{2}{3}\sqrt{80}=5\sqrt{5^3}+6\sqrt{3^2\cdot 5}-7\sqrt{2^2\cdot 5}+\frac{2}{3}\sqrt{2^4\cdot 5}=25\sqrt{5}+18\sqrt{5}-14\sqrt{5}+$$

$$+\frac{2\cdot 2^2}{3}\sqrt{5}=29\sqrt{5}+\frac{8}{3}\sqrt{5}=\left(\frac{87+8}{3}\right)\cdot\sqrt{5}=\frac{95}{3}\sqrt{5}$$

$$2a\sqrt{2}-\sqrt{8}+3\sqrt{2}=2a\sqrt{2}-\sqrt{2^3}+3\sqrt{2}=2a\sqrt{2}-2\sqrt{2}+3\sqrt{2}=2a\cdot\sqrt{2}+\sqrt{2}=\sqrt{2}\cdot(2a+1)$$

$$2a\sqrt{3}-\sqrt{27a^2}+a\sqrt{12}=2a\sqrt{3}-\sqrt{3^3\cdot a^2}+a\sqrt{3\cdot 2^2}=2a\sqrt{3}-3a\sqrt{3}+2a\sqrt{3}=a\sqrt{3}$$

6.- Introduce los factores dentro de cada raíz:

$$2\cdot\sqrt{3}=\sqrt{3\cdot 2^2}=\sqrt{12}$$

$$5\cdot\sqrt{5}=\sqrt{5\cdot 5^2}=\sqrt{125}$$

$$2\cdot\sqrt[3]{5}=\sqrt[3]{5\cdot 2^3}=\sqrt[3]{40}$$

$$2\cdot\sqrt[3]{\frac{5}{2}}=\sqrt[3]{\frac{5\cdot 2^3}{2}}=\sqrt[3]{\frac{40}{2}}=\sqrt[3]{20}$$

$$\frac{1}{4}\cdot\sqrt[3]{\frac{5}{2}}=\sqrt[3]{\frac{5\cdot 1^3}{4^3}}=\sqrt[3]{\frac{5}{64}}$$

7.- Extrae de la raíz el factor que puedas

$$\sqrt[3]{16}=\sqrt[3]{2^4}=2\cdot\sqrt[3]{2}$$

$$4\sqrt{9}=4\cdot\sqrt{3^2}=4\cdot 3=12$$

$$\sqrt[5]{64}=\sqrt[5]{2^6}=2\cdot\sqrt[5]{2}$$

$$\sqrt{\frac{27}{4}}=\sqrt{\frac{3^3}{2^2}}=\frac{3}{2}\cdot\sqrt{3}$$

$$\sqrt[5]{\frac{5x^{10}}{y^8}}=\frac{x^2}{y}\sqrt[5]{\frac{5}{y^3}}=\frac{x^2}{y}\cdot\frac{\sqrt[5]{5}}{\sqrt[5]{y^3}}=\frac{x^2}{y}\cdot\frac{\sqrt[5]{5}\cdot\sqrt[5]{y^2}}{\sqrt[5]{y^3}\cdot\sqrt[5]{y^2}}=\frac{x^2\cdot\sqrt[5]{5y^2}}{y^2}$$

$$\sqrt[3]{\frac{8x^4y^3z}{n^6}} = \sqrt[3]{\frac{2^3\cdot x^4\cdot y^3\cdot z}{n^6}} = \frac{2xy}{n^2}\sqrt[3]{xz}$$

$$\sqrt[4]{\frac{32x^6}{81y^5}} = \sqrt[3]{\frac{2^5\cdot x^6}{3^4\cdot y^5}} = \frac{2x^2}{3y}\cdot\sqrt[3]{\frac{2^2}{3y^2}}$$

8a.- Racionalizar:

a) $\frac{5}{\sqrt{3}} = \frac{5\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}\frac{5\sqrt{3}}{\sqrt{3^2}} = \frac{5\sqrt{3}}{3}$

b) $\frac{2}{\sqrt{8}} = \frac{2}{\sqrt{2^3}}\frac{2}{2\cdot\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$

c) $\frac{5}{\sqrt[3]{5}} = \frac{5\cdot\sqrt[3]{5^2}}{\sqrt[3]{5}\cdot\sqrt[3]{5^2}}\frac{5\sqrt[3]{5^2}}{\sqrt[3]{5^3}} = \frac{5\cdot\sqrt[3]{25}}{5} = \sqrt[3]{25}$

d) $\frac{2}{\sqrt[3]{9}} = \frac{2\sqrt[3]{3}}{\sqrt[3]{3^2}\cdot\sqrt[3]{3}}\frac{2\cdot\sqrt[3]{3}}{\sqrt[3]{3^3}} = \frac{2\cdot\sqrt[3]{3}}{3} = \frac{2\cdot\sqrt[3]{3}}{3}$

e) $\frac{4}{\sqrt{18}} = \frac{4}{\sqrt{2\cdot 3^2}} = \frac{4\cdot\sqrt{2}}{3\cdot\sqrt{2}\cdot\sqrt{2}}\frac{4\cdot\sqrt{2}}{3\cdot\sqrt{2^2}} = \frac{4\cdot\sqrt{2}}{3\cdot 2} = \frac{2\cdot\sqrt{2}}{3}$

f) $\frac{2}{\sqrt[3]{100}} = \frac{2\cdot\sqrt[3]{100^2}}{\sqrt[3]{100}\cdot\sqrt[3]{100^2}}\frac{2\sqrt[3]{100^2}}{\sqrt[3]{100^3}} = \frac{2\cdot\sqrt[3]{100^2}}{100} = \frac{\sqrt[3]{100^2}}{50}$

g) $\frac{4}{\sqrt{50}} = \frac{4\cdot\sqrt{50}}{\sqrt{50}\cdot\sqrt{50}} = \frac{4\sqrt{50}}{\sqrt[2]{50^2}} = \frac{4\cdot\sqrt{50}}{50} = \frac{\sqrt{50}}{25} = \frac{\sqrt{5^2\cdot 2}}{5^2} = \frac{5\cdot\sqrt{2}}{5^2} = \frac{\sqrt{2}}{5}$

h) $\frac{5}{\sqrt[3]{25}} = \frac{5\cdot\sqrt[3]{5}}{\sqrt[3]{5^2}\cdot\sqrt[3]{5}}\frac{5\cdot\sqrt[3]{5}}{\sqrt[3]{5^3}} = \frac{5\cdot\sqrt[3]{5}}{\sqrt[3]{5^3}} = \frac{5\cdot\sqrt[3]{5}}{5} = \sqrt[3]{5}$

.

$$i)\ \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{(\sqrt{2}-1)\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}\cdot\sqrt{2}-\sqrt{2\cdot 1}}{\sqrt{2^2}} = \frac{\sqrt{2^2}-\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2} = 1-\frac{\sqrt{2}}{2}$$

$$j)\ \frac{2\cdot\sqrt{3}-\sqrt{2}}{\sqrt{18}} = \frac{(2\cdot\sqrt{3}-\sqrt{2})\cdot\sqrt{18}}{\sqrt{18}\cdot\sqrt{18}} = \frac{2\sqrt{3}\cdot\sqrt{18}-\sqrt{2}\cdot\sqrt{18}}{\sqrt{18^2}} = \frac{2\cdot\sqrt{2\cdot 3^3}-\sqrt{2^2\cdot 3^2}}{18} = \frac{6\cdot\sqrt{6}-6}{18} =$$

$$= \frac{\sqrt{6}-1}{3}$$

8b.-Racionalizar:

*El conjugado del denominador de un binomio es igual al mismo binomio con el signo central cambiado de signo.

$$a+b \rightarrow a-b$$
$$-a+b \rightarrow -a-b$$
$$a-b \rightarrow a+b$$
$$-a+b \rightarrow -a-b$$

$$a)\ \frac{5}{\sqrt{3}+1} = \frac{5\cdot(\sqrt{3}-1)}{(\sqrt{3}+1)\cdot(\sqrt{3}-1)} \frac{5\cdot(\sqrt{3}-1)}{\sqrt{3^2}-1^2} = \frac{5\cdot(\sqrt{3}-1)}{3-1} = \frac{5\cdot(\sqrt{3}-1)}{2}$$

$$b)\ \frac{2}{\sqrt{2}+2} = \frac{2\cdot(\sqrt{2}-2)}{(\sqrt{2}+2)\cdot(\sqrt{2}-2)} = \frac{2\cdot\sqrt{2}-2\cdot 2}{\sqrt{2^2}-2^2} = \frac{2\cdot\sqrt{2}-4}{2-4} = \frac{2\cdot(\sqrt{2}-2)}{-2} = -(\sqrt{2}-2) = 2-\sqrt{2}$$

$$c)\ \frac{11}{\sqrt{5}+3} = \frac{11\cdot(\sqrt{5}-3)}{(\sqrt{5}+3)\cdot(\sqrt{5}-3)} = \frac{11\cdot\sqrt{5}-11\cdot 3}{\sqrt{5^2}-3^2} = \frac{11\cdot\sqrt{5}-33}{5-9} = \frac{11\cdot\sqrt{4}-33}{-4} = \frac{-11\cdot(\sqrt{4}-3)}{4}$$

$$d)\ \frac{11}{\sqrt{5}+3} = \frac{11\cdot(\sqrt{5}-3)}{(\sqrt{5}+3)\cdot(\sqrt{5}-3)} = \frac{11\cdot\sqrt{5}-11\cdot 3}{\sqrt{5^2}-3^2} = \frac{11\cdot\sqrt{5}-33}{5-9} = \frac{11\cdot\sqrt{4}-33}{-4} = \frac{-11\cdot(\sqrt{4}-3)}{4}$$

$$e)\ \frac{b-2}{\sqrt{b}-2} = \frac{(b-2)\cdot(\sqrt{b}+2)}{(\sqrt{b}-2)\cdot(\sqrt{b}+2)} \frac{b\cdot\sqrt{b}+2b-2\sqrt{b}-4}{\sqrt{b^2}-2^2} = \frac{(b-2)\cdot\sqrt{b}+2b-4}{b-2} = \sqrt{b}+2b-4$$

.

e) $$\frac{x-y}{\sqrt{x}+\sqrt{y}}=\frac{(x-y)\cdot\left(\sqrt{x}-\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}=\frac{(x-y)\cdot\left(\sqrt{x}-\sqrt{y}\right)}{x-y}=\sqrt{x}-\sqrt{y}$$

f) $$\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}-1}+\frac{1}{\sqrt{2}+1}=\frac{1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}+\frac{1\cdot\left(\sqrt{2}+1\right)}{\left(\sqrt{2}-1\right)\cdot\left(\sqrt{2}+1\right)}+\frac{1\cdot\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}-1\right)}=$$

$$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}+1}{\sqrt{2^2}-1^2}+\frac{\sqrt{2}+1}{\sqrt{2^2}-1^2}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}+1}{1}+\frac{\sqrt{2}+1}{1}=\frac{\sqrt{2}}{2}+2\cdot\left(\sqrt{2}+1\right)=$$

$$\frac{\sqrt{2}}{2}+2\sqrt{2}+2=\frac{\sqrt{2}+4\sqrt{2}+4}{2}=\frac{5\sqrt{2}+4}{2}=\frac{5\sqrt{2}}{2}+2$$

g) $$\frac{1}{\sqrt{x}+\sqrt{y}}+\frac{1}{\sqrt{x}+\sqrt{y}}=\frac{2}{\sqrt{x}+\sqrt{y}}=\frac{2\cdot\left(\sqrt{x}-\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}=\frac{2\cdot\left(\sqrt{x}-\sqrt{y}\right)}{\sqrt{x}^2-\sqrt{y}^2}=$$

$$=\frac{2\cdot\left(\sqrt{x}-\sqrt{y}\right)}{x-y}$$

h)

$$\frac{5\sqrt{2}-3\sqrt{3}}{5\sqrt{2}+3\sqrt{3}}=\frac{\left(5\sqrt{2}-3\sqrt{3}\right)\cdot\left(5\sqrt{2}-3\sqrt{3}\right)}{\left(5\sqrt{2}+3\sqrt{3}\right)\cdot\left(5\sqrt{2}-3\sqrt{3}\right)}=\frac{\left(5\sqrt{2}-3\sqrt{3}\right)^2}{\left(5\sqrt{2}\right)^2-\left(3\sqrt{3}\right)^2}=\frac{50-\ 2\cdot5\cdot3\cdot\sqrt{2\cdot3}-27}{25\cdot2-9\cdot3}=\frac{23-45\sqrt{6}}{23}$$

i) $$\frac{6\cdot(3-y)}{\sqrt[3]{(3-y)^2}}=\frac{6\cdot(3-y)\sqrt[3]{(3-y)}}{\sqrt[3]{(3-y)^2}\cdot\sqrt[3]{(3-y)}}=\frac{6\cdot(3-y)\sqrt[3]{(3-y)}}{\sqrt[3]{(3-y)^3}}=\frac{6\cdot\cancel{(3-y)}\sqrt[3]{(3-y)}}{\cancel{3-y}}=6\sqrt[3]{(3-y)}$$